A Proofs for Main Paper

Proof of Theorem 4.1. Proceed by case analysis on δ. Assume validity of δ as described in Section 4.1.

(1) $\delta = \text{setChannel}(\text{id}^\star, C, \text{fn})$. Let $\langle T, D, S\rangle = P[\text{id}^\star]$. Then

$$\begin{aligned}
\mathbf{eval}(\mathbf{patch}(\delta, P)) &= \mathbf{eval}([\text{id}^\star \mapsto \langle T, D, [C \mapsto \text{fn}]S\rangle]P) \\
&= \mathbf{evalLayer}(\text{id}^\star, \langle T, D, [C \mapsto \text{fn}]S\rangle) \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{getMT}(T), \text{id}^\star, \text{d}, [C \mapsto \text{fn}]S) \mid \text{d} \in D\} \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, [C \mapsto \text{fn}]S) \mid \mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S) \in \mathbf{evalLayer}(\text{id}^\star, \langle T, D, S\rangle)\} \\
&\qquad \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, [C \mapsto \text{fn}]S) \mid \mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S) \in \mathbf{eval}(P)\} \\
&\qquad \cup \{\mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \mid \mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \in \mathbf{eval}(P), \text{id} \neq \text{id}^\star\} \\
&= \mathbf{recon}(\delta, \mathbf{eval}(P))
\end{aligned}$$

(2) $\delta = \text{removeChannel}(\text{id}^\star, C)$. Let $\langle T, D, S\rangle = P[\text{id}^\star]$. Then

$$\begin{aligned}
\mathbf{eval}(\mathbf{patch}(\delta, P)) &= \mathbf{eval}([\text{id}^\star \mapsto \langle T, D, S \setminus \{C\}\rangle]P) \\
&= \mathbf{evalLayer}(\text{id}^\star, \langle T, D, S \setminus \{C\}\rangle) \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{getMT}(T), \text{id}^\star, \text{d}, S \setminus \{C\}) \mid \text{d} \in D\} \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S \setminus \{C\}) \mid \mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S) \in \mathbf{evalLayer}(\text{id}^\star, \langle T, D, S\rangle)\} \\
&\qquad \cup \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S \setminus \{C\}) \mid \mathbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S) \in \mathbf{eval}(P)\} \\
&\qquad \cup \{\mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \mid \mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \in \mathbf{eval}(P), \text{id} \neq \text{id}^\star\} \\
&= \mathbf{recon}(\delta, \mathbf{eval}(P))
\end{aligned}$$

(3) $\delta = \text{addLayer}(\text{id}^\star, \ell)$. Then

$$\begin{aligned}
\mathbf{eval}(\mathbf{patch}(\delta, P)) &= \mathbf{eval}([\text{id}^\star \mapsto \ell]P) \\
&= \mathbf{evalLayer}(\text{id}^\star, \ell) \cup \mathbf{eval}(P) \\
&= \mathbf{recon}(\delta, \mathbf{eval}(P))
\end{aligned}$$

(4) $\delta = \text{removeLayer}(\text{id}^\star)$. Then

$$\begin{aligned}
\mathbf{eval}(\mathbf{patch}(\delta, P)) &= \mathbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \bigcup_{\text{id} \in P, \text{id} \neq \text{id}^\star} \mathbf{evalLayer}(\text{id}, P[\text{id}]) \\
&= \{\mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \mid \mathbf{mark}(\text{mt}, \text{id}, \text{d}, S) \in \mathbf{eval}(P), \text{id} \neq \text{id}^\star\} \\
&= \mathbf{recon}(\delta, \mathbf{eval}(P))
\end{aligned}$$

The third equality holds because marks from **evalLayer** are guaranteed to have the same id as the layer they are evaluated from.

(5) $\delta = \text{transformLayer}(\text{id}^\star, \text{transform})$. Let $\langle T, D, S\rangle = P[\text{id}^\star]$. Let $\text{transform}(T, D, S) = \langle T', D', S'\rangle$. Then

$$\begin{aligned}
\textbf{eval}(\textbf{patch}(\delta, P)) &= \textbf{eval}([\text{id}^\star \mapsto \langle T', D', S'\rangle]P) \\
&= \textbf{evalLayer}(\text{id}^\star, \langle T', D', S'\rangle) \cup \textbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \{\textbf{mark}(\text{getMT}(T'), \text{id}^\star, \text{d}', S') \mid \text{d}' \in D'\} \cup \textbf{eval}(P \setminus \{\text{id}^\star\}) \\
&\quad\ \cup \textbf{eval}(P \setminus \{\text{id}^\star\}) \\
&= \Big\{\textbf{mark}(\text{getMT}(T'), \text{id}^\star, \text{d}', S') \mid \textbf{mark}(\text{mt}, \text{id}^\star, \text{d}, S) \in \textbf{eval}(P), \\
&\qquad\quad \text{transform}_1(\text{getMT}^{-1}(\text{mt}), \text{d}, S) = \langle T', \text{d}', S'\rangle\Big\} \\
&\quad\ \cup \{\textbf{mark}(\text{mt}, \text{id}, \text{d}, S) \mid \textbf{mark}(\text{mt}, \text{id}, \text{d}, S) \in \textbf{eval}(P), \text{id} \neq \text{id}^\star\} \\
&= \textbf{recon}(\delta, \textbf{eval}(P))
\end{aligned}$$

The fourth equality holds because transform maps transform_1 over D.

□

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